A Gossip-Based Approach for Measurement Task Allocation and Routing in Multi-Robot Systems with Heterogeneous Sensing

Hamza Chakraa, **Diego Deplano**, Carla Seatzu, Dimitri Lefebvre, and Mauro Franceschelli

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1. Problem statement and contributions
2. MILP formulation and the proposed gossip heuristic
3. Numerical simulations and conclusions

Problem of interest

The problem: Managing a multi-robot team responsible for performing measurement tasks across a shared environment

The objective: Minimizing the completion time while meeting robots energy capacity

Our contribution: A gossip-based heuristic to compute (or improve) off-line solutions and make them near-optimal up to pairwise exchange between robots

Three-folded challenge:

- Task assignment
- Route planning
- Execution scheduling

Problem of interest

Main differences with the standard MVRP (multi-veichle routing problem):

- Each robot has on board a different set of sensors
- Each robot can execute more measurements simultaneously at the same location
- Each robot has a limited operational capacity (e.g., battery)

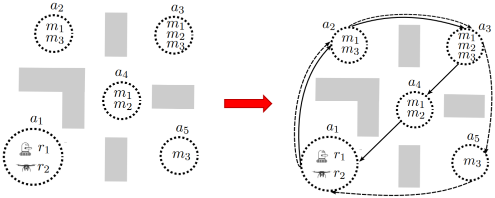


Figure 1: A simple example

Tasks, robots, and objective functions

Modelling

- $\mathscr{R} = \{r_1, \ldots, r_k, \ldots, r_R\}$ $\implies$ The set of robots
- $\mathscr{A} = \{a_1, \ldots, a_i, \ldots, a_j, \ldots, a_A\}$ $\implies$ The set of tasks' location
- $\mathscr{M} = \{m_1, \ldots, m_q, \ldots, m_M\}$ $\implies$ The set of measurements
- $\mathscr{J} = \{j_1, \ldots, j_t, \ldots, j_T\} \subseteq \mathscr{A} \times \mathscr{M}$ $\implies$ The set of tasks

Problem inputs

- $t_{qi} \in \{0, 1\}$ $\implies$ measurement m_q should be taken at site a_i if $t_{qi} = 1$
- $p_{kq} \in \{0, 1\}$ $\implies$ robot r_k can perform measurement m_q if $p_{kq} = 1$
- $c(i, j, k) \in \mathbb{R}_{\geq 0}$ $\implies$ travel cost between a_i and a_j
- $b_k \in \mathbb{R}_{\geq 0}$ $\implies$ maximum cost for robot r_k

Mixed-Integer Linear Programming (MILP) model

Decision variables

- $x_{ij}^k \in \{0,1\} \implies$ robot r_k moves from location a_i to a_j
- $u_i^k \in \mathbb{N} \implies$ order of visit of site a_i by robot r_k

Objective functions

- $$\min \max_{k \in \mathscr{R}} \sum_{i,j \in \mathscr{A}} x_{ij}^k \cdot c(i,j,k)$$

  $\implies$ Maximal travelling cost over all robots

Mixed-Integer Linear Programming (MILP) model

Constraint 1: Robots are moving around oriented tours (circuits)

- $\sum_{j \in \mathcal{A}} x_{1j}^k = 1 \qquad \forall k \in \mathcal{R}$

  $\implies$ The mission of each robot r_k starts from the depot a_1

- $\sum_{j \in \mathcal{A}} x_{ij}^k = \sum_{j \in \mathcal{A}} x_{ji}^k \qquad \forall k \in \mathcal{R},\ \forall i \in \mathcal{A}$

  $\implies$ Each agent r_k leaves any site a_i as many times as it enters it

- $u_i^k + x_{ij}^k \leq u_j^k + (A-1) \cdot (1 - x_{ij}^k) \qquad \forall k \in \mathcal{R},\ \forall i \in \mathcal{A},\ \forall j \in \mathcal{A} / \{a_1\}$

  $\implies$ Sub-tours elimination constraint

Mixed-Integer Linear Programming (MILP) model

Constraint 2: Tasks must be performed by suitable robots

- $\sum_{k \in \mathcal{R}} \sum_{i \in \mathcal{A}} p_{kq} \cdot x_{ij}^{k} \geq t_{qj} \qquad \forall q \in \mathcal{M},\ \forall j \in \mathcal{A}$

  $\Longrightarrow$ At least one robot able to perform m_q should visit a_j if m_q is required at a_j

Constraint 3: Each robot have a finite reserve of autonomy

- $\sum_{i,j \in \mathcal{A}} x_{i,j}^{k} \times c(i,j,k) \leq b_k \qquad \forall r_k \in \mathcal{R}$

  $\Longrightarrow$ Robots have a finite reserve of time or energy

The proposed gossip-based heuristic

Decentralized Gossip Heuristic

Algorithm 1 Gossip Heuristic

```
If not given: Compute initial task sequences for each robot.
Compute possible_pairs: robot pairs with common sensors.
F = 1
while F = 1 do
    F = 0
    Shuffle possible_pairs randomly.
foreach pair in possible_pairs
    Apply Algorithm 2 on pair.
    if solution improved then
        Set F = 1
    end if
end while
```

Task Exchange Mechanism

Algorithm 2 Task Exchange for r_k, r_q

Require: Task sequences $\mathscr{S}(k)$, $\mathscr{S}(q)$ for robots r_k, r_q

- **Assume:** $\mathscr{C}(\mathscr{S}(q)) < \mathscr{C}(\mathscr{S}(k))$
- $\mathscr{J}_{ex}$: tasks of r_k that r_q can perform.
- **while** $\mathscr{J}_{ex} \neq \emptyset$ **do**
  - Select $t_j \in \mathscr{J}_{ex}$ randomly
  - $\mathscr{J}_{ex} = \mathscr{J}_{ex} \setminus \{t_j\}$
  - $\mathscr{S}_{new}(q)$ = new solution for robot r_q with $\mathscr{S}(q) \cup \{t_j\}$
  - **if** $\mathscr{C}(\mathscr{S}_{new}(q)) < \mathscr{C}(\mathscr{S}(k))$ **then**
    - Set $\mathscr{S}(q) = \mathscr{S}_{new}(q)$, $\mathscr{S}(k) = \mathscr{S}(k) \setminus \{t_j\}$
  - **end if**
- **end while**

Some remarks

Proposition 1

The proposed gossip-based heuristic terminates in a finite number of iterations.

Proposition 2

The worst-case time complexity of one iteration of the proposed gossip-based heuristic is $\mathcal{O}(J \cdot R^2)$, where J is the number of tasks and R the number of robots.

Bounding the optimal solution

The lower bound

- The relaxation of the problem provides an objective value, which serves as a lower bound:

$$0 \le x_{i,j}^k \le 1, \quad \forall r_k \in \mathcal{R}, \forall a_i, a_j \in \mathcal{A}$$
$$u_i^k \in \mathbb{R}^+, \quad \forall r_k \in \mathcal{R}, \forall a_i \in \mathcal{A}$$

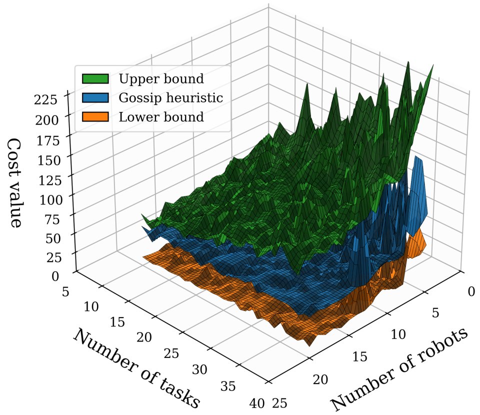
The upper bound

- The estimation of an upper bound was done by employing Monte Carlo simulations.

Numerical simulations

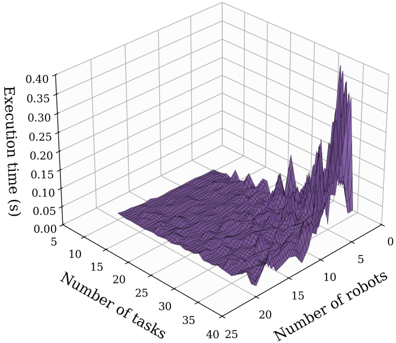
Optimality of the solution

- The upper and lower bounds are computed as the mean over 10 independent experimental runs.
- The cost value obtained through the Gossip-based heuristic lies between the upper and lower bounds.

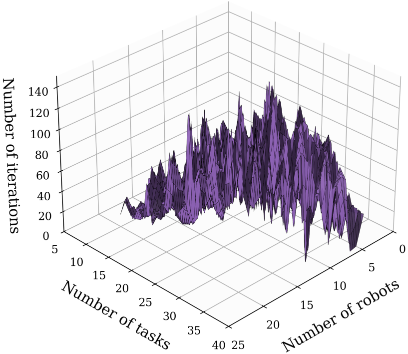


Comparison of the cost value with the upper bound and lower bound across different scenarios

Numerical simulations



Gossip heuristic execution time across several scenarios



Number of iterations required for the Gossip-based heuristic to reach equilibrium across several scenarios

Conclusion & perspectives

Our contribution: A gossip-based heuristic to compute (or improve) off-line solutions and make them near-optimal up to pairwise exchange between robots

Outcomes

- The proposed heuristic systematically improve the given solution
- Monte Carlo simulations demonstrate that the solutions are near optimal
- Low computational time required

Future works

- Compute a theoretically guaranteed upper bound on the quality of the solution
- Test the proposed heuristic in real-time adaptive planning with stochastic delays

Thank you for your attention!